Trumpington Federation calculation policy

Vision

For our children to feel confident, wanting to give it a go, when solving a mathematical problem that they haven't seen before.

Introduction

The purpose of this calculation policy is to ensure consistency and progression in the teaching of maths and the use of different manipulatives, representations and calculation methods across the school from EYFS to Year 6. It aims to give an overview of the key calculation strategies for addition, subtraction, multiplication and division. Our intention is that our children will be flexible with the methods that they use to perform different calculations.

Early Years: for further information on the approach to the teaching of Maths in the Early Years please see the document 'Early Years Approach to Teaching Maths'.

EYFS		
Concrete	Pictorial	Abstract
Objects, Cubes, Counters, Numicon and Fingers Use a range of resources to subitise numbers.	Use pictures to subitise numbers.	
100000000		

Objects, Cubes, Counters, Numicon and Fingers

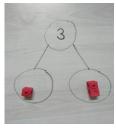
Use a range of resources to automatically recall number bonds, double facts and related subtraction facts.



Five and three makes eight.

OR Eight is made of five and three. Five and three is eight.

Part-whole model







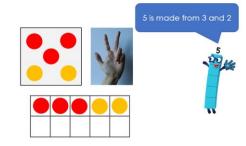
Three and three makes six.

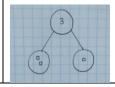
OR Six is made of three and three. Three and three is six.



Five and two makes seven.

Tens frames and Part-whole model





3 and 3 makes 6.

5 is made up of 3 and 2.

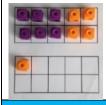
Addition

Year 1

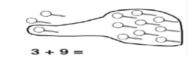
Teur I		
Concrete	Pictorial	Abstract
Cubes and Numicon Use Cubes and Numicon to add two numbers together as a group, on a part-whole model or in a bar.	Use pictures to add two numbers together as a group or in a bar. Part-whole model and Bar model Children are shown how the number sentence can be represented in a part-whole model and on a bar model. Please note that the models can be shown in different orientations.	Use the part-whole diagram or bar model to move into the abstract. 5 + 2 = 7 7 = 5 + 2
Bead strings Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	Number lines Initially, children are taught to start at the biggest number. As they grow in confidence, children start to explore starting with the smallest number to develop an understanding of commutativity.	Place the larger number in your head and count on the smaller number to find your answer. 5 + 2 = 7 7 = 5 + 2
10s frame and Bead strings	5 + 2 = 7 Number lines Use pictures or a number line. Regroup or partition the smaller number using the part-	If I am at seven, how many more do I need to make 10. How many more do I add on now?

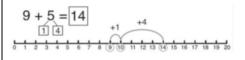
Start with the bigger number and use the smaller number to make 10 using bead strings and 10s frames.





whole model to make 10.

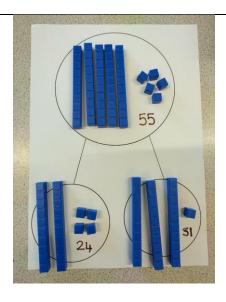


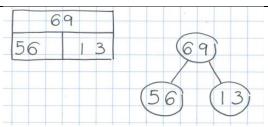


7 + 4 = 11

Year 2

Concrete	Pictorial	Abstract
Dienes and Bead strings		Move into the abstract.
Model using Dienes and Bead strings.	3 tens + 5 tens = tens	56 + 13 = 69
	30 + 50 =	13 + 56 = 69
	Use representations for Dienes.	
// · / / / / / / / / / / / / / / / / /	Jack has 22 sweets. Here are Sophie's sweets:	69 = 56 + 13
Part-whole model with Dienes	How many sweets do they have altogether?	69 = 13 + 56
	now many sweets do they have drogether.	
	Part-whole model and Bar model Children are shown how the number sentence can be represented in a part-whole model and on a bar model.	

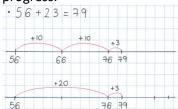




Number lines

Switch to a partially empty number line (e.g. with multiples of 5, 10).

As children become more confident with their number bonds their number line strategies will progress.

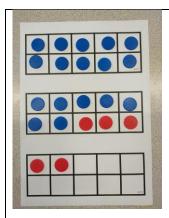


10s frame

Use 10s frames to make 10

Part-whole model and Number line

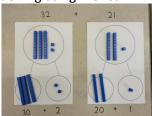
Use part-whole model and number line to model.



CONCRETE

Dienes

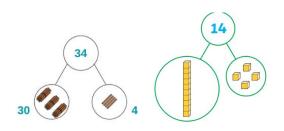
Model partitioning using Dienes.



PICTORIAL

Partitioning

Children are shown how to partition numbers into tens and ones and then add them.



ABSTRACT

Partitioning

Children are shown how to partition numbers into tens and ones and then add them.

Expanded column method

Building on from partitioning, children are shown how to set up the 10s and 1s in columns. They are taught to add the 1s first.

٠	5	6	+	1	3	=	6	9	
	5	0		+		6			
+	1	0		+		3			
	6	0		+		9			

Here is an example where the 1s exceed 9.

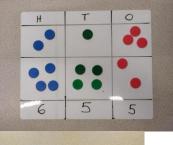


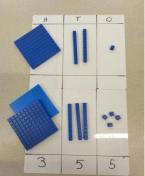
Year 3

Concrete

Dienes and Counters

Model using Dienes and Counters. Add together the ones first then the 10s then the 100s.





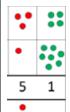
Pictorial

Place value chart

Children move to drawing the counters using a place value chart with 100s, tens and ones.

Т	0	<u>@</u>	<u> </u>	•
			0000	0000
			0	00000
tens	ones			

Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.



Abstract

Expanded column method

Children are shown how to set up the 100s, 10s and 1s in columns, showing the full value of each digit. They are taught to add the 1s first.

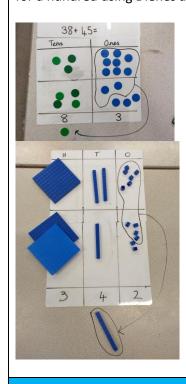
3	5	6	+	.2	1	3	=	5	6	9
	3	0	0	+	5	0	+	6		
	2	0	0	+	1	0	+	3		
	5	0	0	+	6	0	+	9		

3	6	6	. +	2	6	3	=	6	2	9	
		0		+	6	0	+	6			
	2	0	0	+	6	0	+	3			
_	6	0	0	+	2	0	+	9	<u>-</u> .;;		

Compact column method

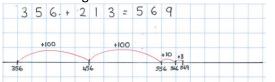
Building on from the expanded column method, children are shown how to set up an addition by putting their 100s, 10s and 1s in the correct place value column. They are taught to add the 1s first.

Model exchanging 10 ones for a ten and 10 tens for a hundred using Dienes and Counters.



Number lines

Switch to an empty number line to increase flexibility and speed and allow demonstration of understanding.



Children are shown how the number sentence can be represented in a part-whole model and on a bar model.

(5 6 9)			5	6	9				_
	3	5	6		İ	2	J	3	
(356) (213)									

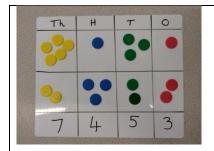
3	5	6	+	2	١	3	Ξ	5	6	9	
	3	5	63								
	2	1	3								
	5	6	9								

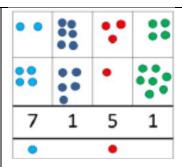
With exchanging

0					
7					
5					
=	-				
3					
1					
2		7	3	0	
+		5	1	7	1
7		3	2	5	
5			Ť		
3					

Year 4

Concrete	Pictorial	Abstract
Counters Children continue to use Counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.	Place value chart Children draw representations using a Place value chart.	Children consolidate their learning from previous years. They continue to be shown how to use the expanded and the compact column method. [Children to be shown how to use the compact column method when adding decimals]





Children consolidate their learning from previous years.

They continue to be shown how to use a blank number line and how the number sentence can be represented in a part-whole model and on a bar model.

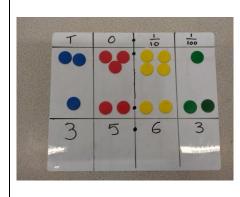
[Children to be shown how to use the number line when adding decimals]

Year 5

Concrete

Counters

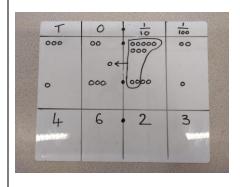
Include decimal Counters and model exchange for addition.



Pictorial

Place value chart

Children draw representations using a Place value chart.



Abstract

Children consolidate their learning from previous years.

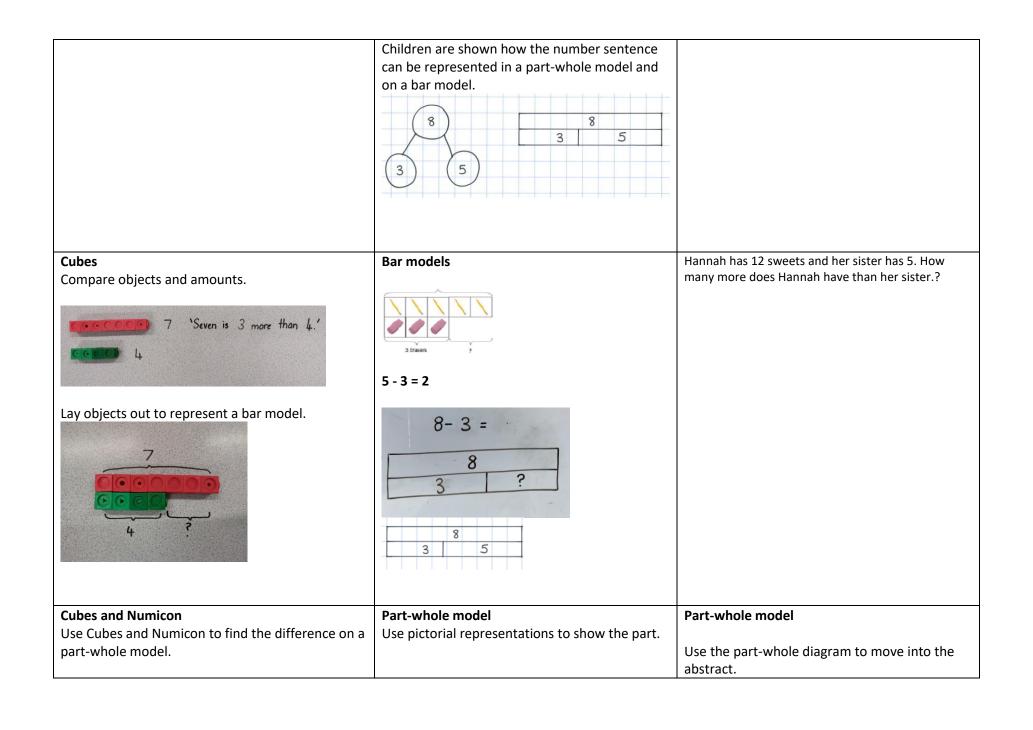
They continue to be shown how to use the expanded and the compact column method.

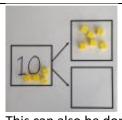
[Children to be shown how to use the compact column method when adding decimals]

	Children consolidate their learning from previous years.	
	They continue to be shown how to use a blank number line and how the number sentence can be represented in a part-whole model and on a bar model. [Children to be shown how to use the number line when adding decimals]	
Year 6		
As Year 5 with increasingly larger and more complex numbers and decimal values.	As Year 5 with increasingly larger and more complex numbers and decimal values.	As Year 5 with increasingly larger and more complex numbers and decimal values.

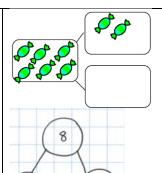
Subtraction

Year 1		
Concrete	Pictorial	Abstract
Physical objects, counters, cubes Use physical objects, counters, cubes etc to show how objects can be taken away.	Cross out drawn objects to show what has been taken away.	15 – 3 = 12
Bead strings	Number lines and part-whole model	8 – 3 = 5
Take away Move the beads along the bead string one by one as you count backwards. $8-3=5$	Take away Count back in ones using a number line. Find the difference Children learn that it is possible to find the difference by counting forwards from one number or counting back from the other	Put 8 in your head, count back 3. What number are you at now?
Find the difference Children learn that it is possible to find the difference by counting forwards from one number or counting back from the other number.	number. 8 - 3 = 5	5 + ? = 8 8 - ? = 5 5 = 8 - ?





This can also be done with Numicon.



8 - 3 = 5

10s frame and Bead strings

Make the number on the 10s frame or bead string then take away to make ten. Then, take the rest away.



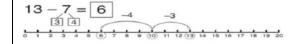
14 - 5 = 9



14 - 4 = 10 10 - 1 = 9

Number line

Jump back 3 first to make 10 and then 4 more. Use ten as the stopping point.



How many do we take off first to get to 10? How many left to take off?

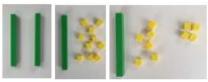
16 – 8 = ?

Year 2

Concrete Pictorial Abstract

Dienes

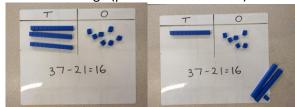
Children are shown how to change a ten into ten 1s and then take away.





Dienes

Children are shown how to make the larger number using Dienes and then take away (remove) the smaller number. Children should also be taught how to exchange (please refer to Year 3).



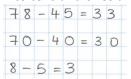




Partitioning

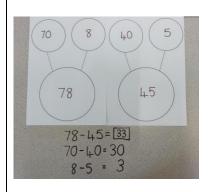
20 - 4 = 16

Children are shown how to partition numbers into tens and ones and then subtract them.



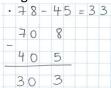
Part-whole model

Children are shown how to partition numbers using a part-whole model into tens and ones and then subtract them.



Expanded column method

Building on from partitioning, children are shown how to set up the 10s and 1s in columns with the larger number on the top. They are taught to subtract the 1s first.

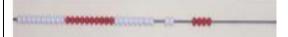


Here is an example with exchanging.

ı	16	216	= 1	S 0	111	С.	ха	111	μı
	•	7	5	-	4	8	=	2	7
			67	00		_1	5 B		
			4	0			8		
			2	0			7		

Bead strings

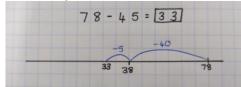
Children are shown how to take away and how to find the difference using bead strings and the next ten strategy.



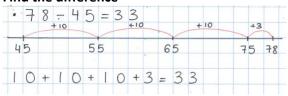
Number lines

Switch to a partially empty number line (e.g. with multiples of 5, 10).

Take away



Find the difference



78 - 45 = 33

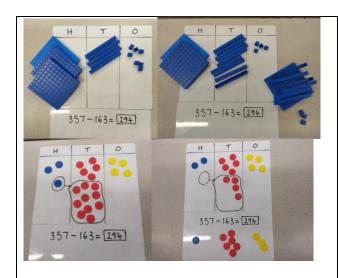
Year 3

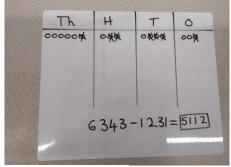
Concrete	Pictorial	Abstract
Dienes and Counters Model using Dienes and Counters and taking away.	Draw representations to support understanding of taking away by crossing out.	54 – 22 - 32
357 - 123 = 234 357 - 123 = 234		
Dienes and Counters	Children to draw or be shown Dienes or	Expanded column method

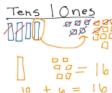
Model exchanging a hundred for 10 10s and ten for 10 1s using Dienes and Counters.

Counters.

Children are shown how to set up the 100s, 10s and 1s in columns, showing the full value of each digit with the larger number on top. They are taught to subtract the 1s first.



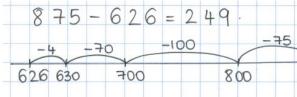




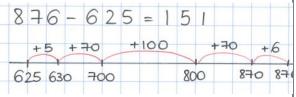
Number lines

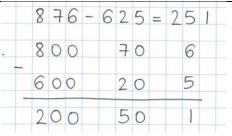
Switch to an empty number line to increase flexibility and speed and allow demonstration of understanding.

Take away



Find the difference



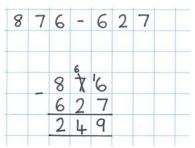


Compact column method

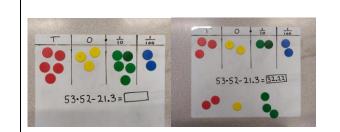
Building on from the expanded column method, children are shown how to set up a subtraction putting their 100s, 10s and 1s in the correct place value column. They are taught to subtract the 1s first.

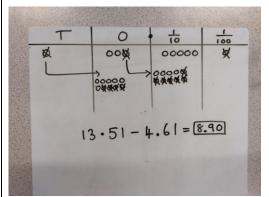
	8	7	6	
_	6	2	5	
	2	5	1	

With exchanging



	Children are shown how the number sentence can be represented in a part-whole model and on a bar model.	
Year 4		
Concrete	Pictorial	Abstract
Counters Children continue to use Counters to subtract, exchanging a thousand for ten 100s and a hundred for ten 10s etc.	Place value chart Children draw representations using a Place value chart and show their exchange. The Horizontal Advance of the shown their exchange of the shown how to use a blank number line and how the number sentence can be represented in a part-whole model and on a bar model. [Children to be shown how to use the number line when subtracting decimals]	Children consolidate their learning from previous years. They continue to be shown how to use the expanded and the compact column method. [Children to be shown how to use the compact column method when subtracting decimals using a zero for place holders]
Year 5		
Concrete	Pictorial	Abstract
Counters Introduce decimal Counters and model exchange for subtraction.	Place value chart Children draw representations using a Place value chart.	Children consolidate their learning from previous years.





Children consolidate their learning from previous years.

They continue to be shown how to use a blank number line and how the number sentence can be represented in a part-whole model and on a bar model.

[Children to be shown how to use the number line when adding decimals]

They continue to be shown how to use the expanded and the compact column method.

[Children to be shown how to use the compact column method when subtracting decimals, using a zero for place holders]

Year 6

As Year 5 with increasingly larger and more complex numbers and decimal values.

As Year 5 with increasingly larger and more complex numbers and decimal values.

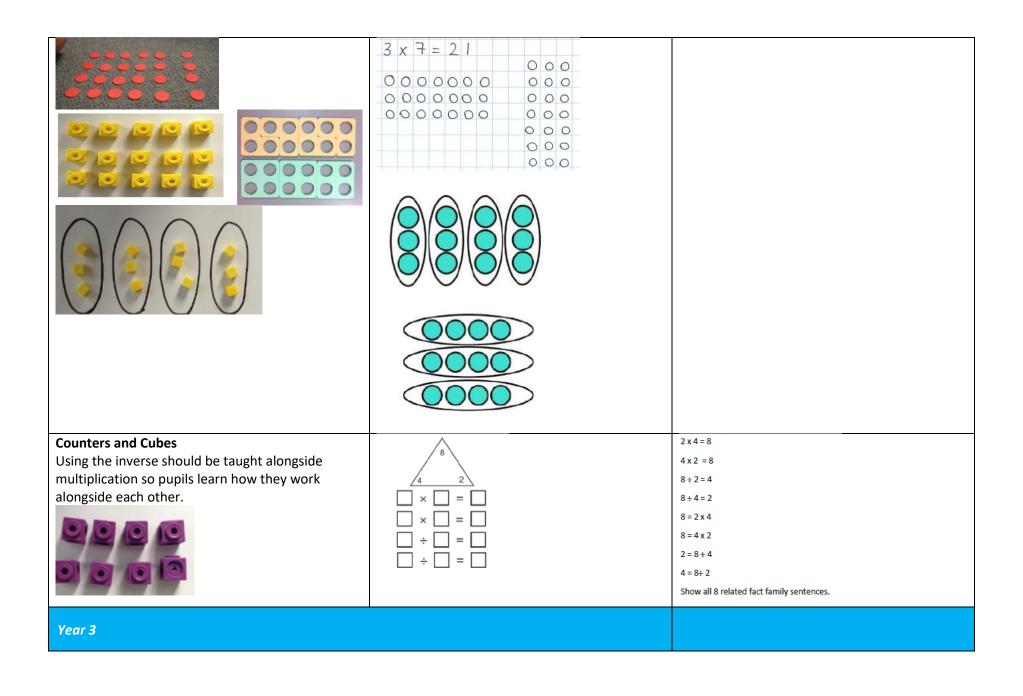
As Year 5 with increasingly larger and more complex numbers and decimal values.

Year 1

Concrete	Pictorial	Abstract
Physical objects, Numicon, Cubes	Number lines	Write addition sentences to describe objects
Use different objects to add equal groups and introduce repeated addition.	Use pictorials including number lines to solve problems.	and pictures.
	There are 3 sweets in one bag. How many sweets are in 5 bags altogether?	
3 + 3 + 3	3+3+3+3 = 15	2 + 2 + 2 + 2 + 2 = 1 0
	2 × 5 = 1 0	
Arrays	Arrays	2 x 5 = 10
Use objects laid out in arrays to find the answers	Draw representations of arrays to show understanding (draw them both ways).	

	2 * 5 = 1 0	
Year 2		

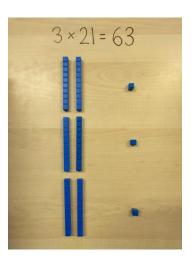
Concrete	Pictorial	Abstract
Bead strings, Cubes and Numicon Multiply using Bead strings, Cubes and Numicon. Continue framing as repeated addition.	Bar models Bar models should be used to show representations of counting in multiples.	3 x 5 = 15 5 x 3 = 15
	My My My My My My	
	3 3 3 3 3	
Arrays	Use representations of arrays to show different calculations and explore commutativity.	3+3+3+3=12
Children are shown how multiplication can be represented as an array.	calculations and explore commutativity.	4 + 4 + 4 = 12
		3 x 4 = 12
		4 x 3 = 12

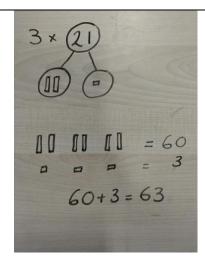


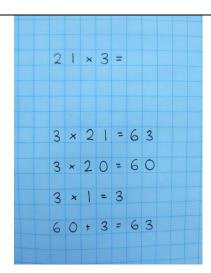
Counters, Dienes and Counters

Partitioning

With smaller numbers, children are shown how to partition numbers into tens and ones and then multiply them.







Counters and Dienes

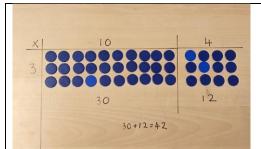
Grid method

Show the link with arrays to first introduce the grid method. Choose sensible numbers and have your model prepared. Children do not need to create the concrete model on their own as it would disrupt the learning. It is important for the children to see the number getting bigger.

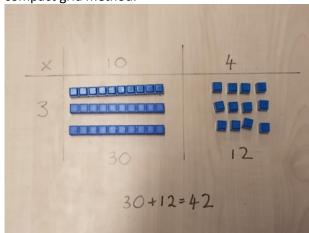
Grid method

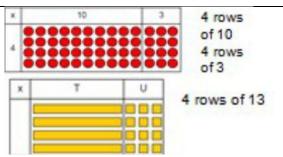
Grid method

Children are shown how to set up the 10s and 1s in a grid and add the products.



Move onto Dienes to move towards a more compact grid method.





Children can represent their work with counters or Dienes in a way that they understand.

They can draw the dienes in different columns to show their thinking as shown below.

Dienes example

	3×24=	
X	20	4
3		0000
		0000
	00	0000
	60	12
	60+1	2=72

× 50 4	6 x	54	11	32	4
	×	5	0	4	
6 300 24	6	3C	0	24	

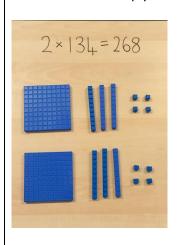
300 + 24 = 324

Year 4

Dienes

Partitioning

With simple 3 digit numbers, children are shown how to partition numbers into hundreds, tens and ones and then multiply them.



2 × (134)
= 200
000 000 = 60
0000 9000 = 8
200+60+8=268
The state of the s

4	Χ	2	7	1	=	1	0	8	4
4	X	2	0	0	-	8	0	0	
4	X	7	0	11	2	8	0		
4	X	1	=	4					

Dienes

Grid method

Recap from Year 3 using Dienes to show how we can multiply using the grid method. We are multiplying by 4 so we need 4 rows of 126.

It is important at this stage that children get into the habit of always multiplying the ones first.

Grid method

Children can represent their work with Dienes to show their thinking.

Dienes example

x 100 30	5 = 1155
----------	----------

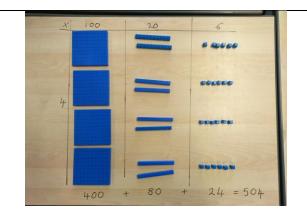
Grid method

Children are shown how to set up the 100s, 10s and 1s in a grid and add the products.

4	x 2	71	=	10	8	4	
	×	200)	7	0		1
	4	800)	28	0		4
	80	0 +	2	80	+	4	

Expanded column method

Building on the grid method, children are shown how to set up a multiplication with the larger



number on the top. Starting with the 1s, they calculate the product of the 1s and the multiplier and follow on from there.

v	2	5	3							
^			6							
		1	8	(6	×	3)		
	3	0	0	(6	×	5	0)	
1	2	0	0	(6	×	2	0	0)
I	5	1	8							

Compact column method

Building on the expanded column method, children are shown how to shorten the method.

	2	5	3	
^			6	
١	5	1	8	
	3	1		

Year 5

Concrete	Pictorial	Abstract

Grid method, Expanded column method and Compact column method

Children consolidate their learning from previous years. They continue to use the grid method, expanded and compact column method for multiplying by a 1 digit number.

Grid method

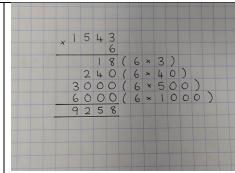
The children revisit how to use the grid method to multiply by a 1 digit number.

Expanded column method

Building on the grid method, children are shown how to set up a multiplication with the larger number on the top. Starting with the 1s, they calculate the product of the 1s and the multiplier and follow on from there.

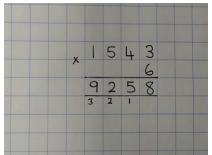
Children can continue to be supported by Counters at this stage of multiplication. This is initially done where there is no regrouping.





Compact column method

Building on the expanded column method, children are shown how to shorten the method.



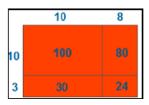
Grid method and Compact column method

Children are shown how to use the grid method and the compact column method to multiply by a 2 digit number.

The numbers are becoming too large to be supported by concrete resources.

Grid method

Children are shown how the grid method can be used when multiplying by a 2 digit number.



Compact column method

Children learn how to use the compact column method multiplying by a 2 digit number.

	1	8		1	2	3	4
×	1	3	×			1	6
	² 5	4		17	24	20	4
_1	8	0	1	2	3	4	0
2	3	4	1	9	7	4	4

Year 6			

Concrete	Pictorial	Abstract
As in Year 5	As in Year 5	As in Year 5

Division

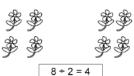
Year 1		
Concrete	Pictorial	Abstract
Physical objects and Cubes Use different objects to show equal sharing of small quantities.	Children use pictures or shapes to share quantities. 8 shared between 2 is 4.	12 shared between 3 is 4. 12 divided by 3 is 4.
Arrays Use objects laid out in arrays to find the answers	Arrays Draw representations of arrays to show understanding (draw them both ways). Use ovals to split the array into groups to aid understanding. 10 shared between 2 is 5. 10 shared between 5 is 2.	10 shared between 2 is 5. 10 shared between 5 is 2. 10 divided by 5 is 2. 10 divided by 2 is 5.
Year 2		

Cubes and Counters

Share quantities into equal groups.



Children use pictures or shapes to share quantities.





 $12 \div 3 = 4$

Bead strings

Divide using repeated subtraction.

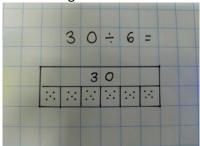
$$35 \div 5 = 7$$

35 grouped into 5s is 7.



Bar model

Divide using a bar model.



This is 30 shared between 6.

 $30 \div 6 = 5$

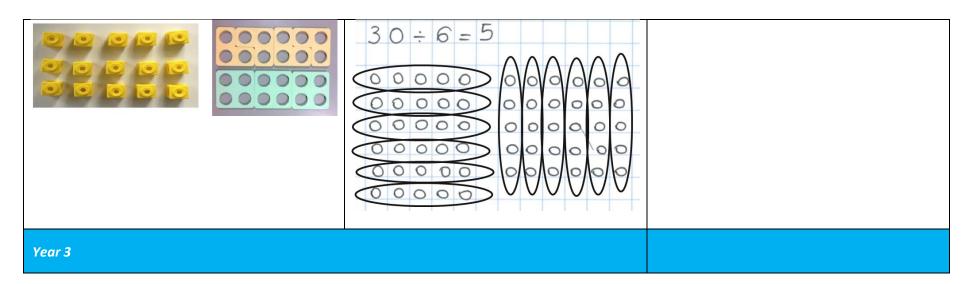
Cubes and counters

Children are shown how division can be represented as an array. Children are shown how to share the dividend by the divisor.

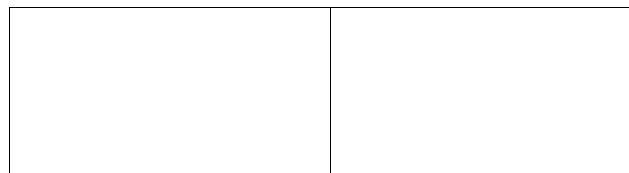
Draw an array and use ovals to split the array into groups to aid understanding.

30 shared between 6 is 5 30 shared between 5 is 6

 $30 \div 6 = 5$ $30 \div 5 = 6$



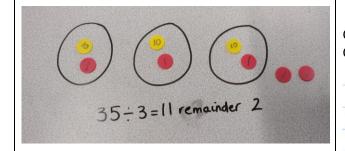
Concrete	Pictorial	Abstract
Cubes and Counters Use Cubes and Counters to aid understanding through sharing.		
96 ÷ 3 = 32 Output Discrete the second of	Draw an array and use ovals to split the array	Find the inverse of multiplication and division
and thinking about the number sentences that can be created.	into groups to make multiplication and division sentences.	sentences by creating eight linking number sentences.
15÷3=5 5x3=15 15÷5=3 3x5=15		



7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Cubes and Counters

Divide Cubes or Counters between groups, without remainders first and then with remainders, seeing how much is left over.



Draw dots and group them to divide an amount and clearly show a remainder.

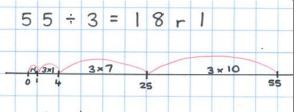








Continue to use grouping and subtraction. Children learn to see how much is left over.



Complete written divisions and show the remainder using r.

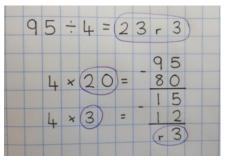
$$29 \div 8 = 3 \text{ REMAINDER 5}$$
 $\uparrow \qquad \uparrow \qquad \uparrow$

dividend divisor quotient remainder

Chunking

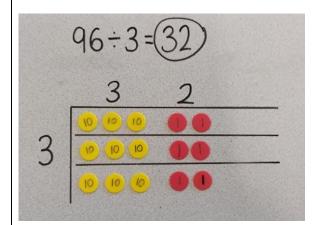
Continue to use grouping and subtract from the dividend.

Children learn to see how much is left over.

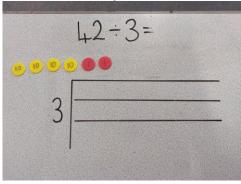


Counters

Use Counters to divide using the bus stop method alongside, starting with no remainders.

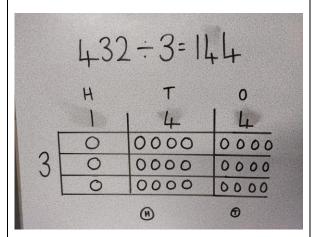


Move onto dividing with remainders.



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

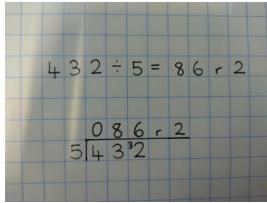
Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Children consolidate their learning from previous years. They continue to use the number line with grouping.

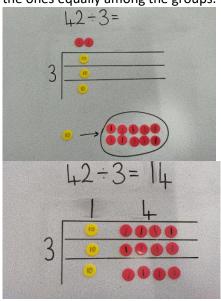
Begin with divisions that divide equally with no remainder.

Move onto divisions with a remainder.



L	+2÷3=
(0)	••
3	10

We exchange this ten for ten ones and then share the ones equally among the groups.



We look at how much is in 1 group so the answer is 14.

Year 5

Concrete	Pictorial	Abstract

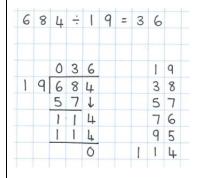
Children consolidate their learning from previous years. They continue to use the number line with grouping and the bus stop method.

Concrete Pictorial Abstract

Children consolidate their learning from previous years. They continue to use the number line with grouping and the bus stop method. These methods can be used to divide by a 2-digit number.

Long division

Children learn to apply this formal method.



Other useful documents:

- Federation approach to teaching times tables
- Mathematical vocabulary progression